Spin, superconductivity, and the Bogoliubov transformation

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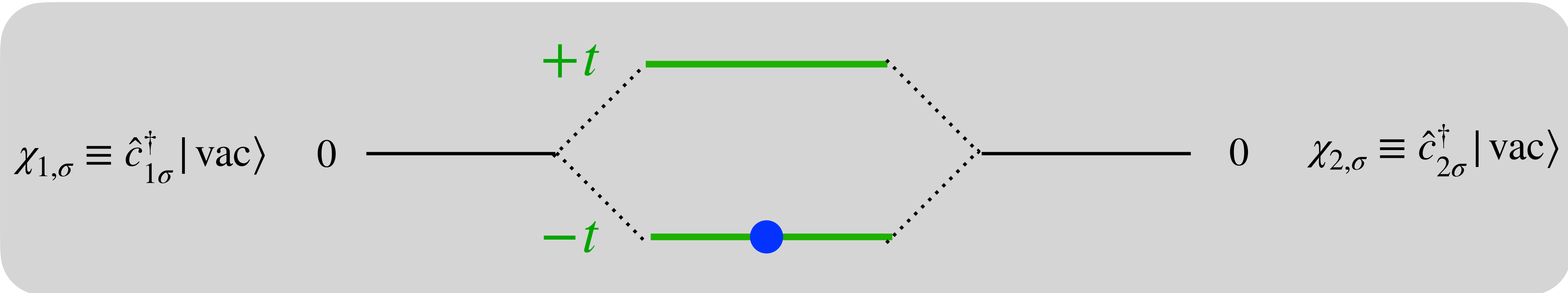
One-electron tight-binding (Hückel) dimer

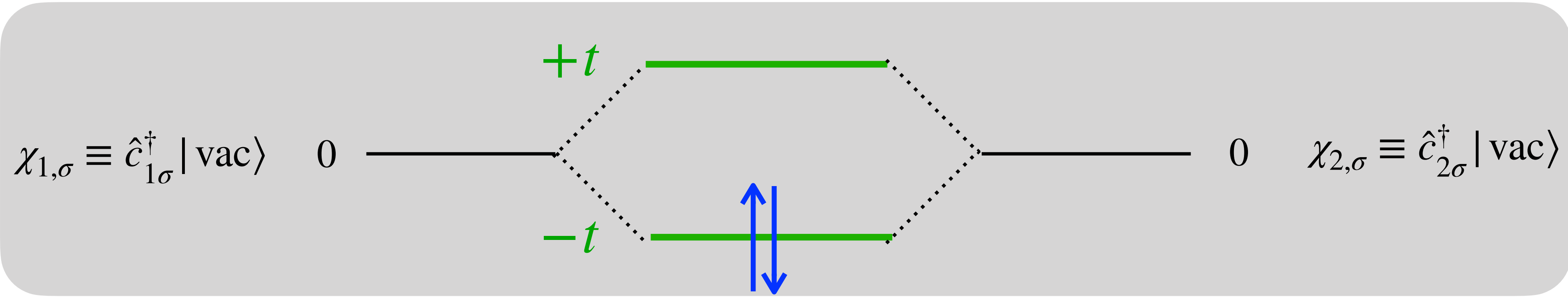
$$\hat{h} = -t \sum_{\sigma=\uparrow,\downarrow} \left(\hat{c}^{\dagger}_{1\sigma}\hat{c}_{2\sigma} + \hat{c}^{\dagger}_{2\sigma}\hat{c}_{1\sigma} \right) = \hat{h}^{\dagger}$$



$$|\Psi_0\rangle = \frac{1}{\sqrt{2}}\left(\hat{c}^{\dagger}_{1\sigma} + \hat{c}^{\dagger}_{2\sigma}\right)|\mathrm{vac}\rangle \quad \rightarrow \quad \hat{h}|\Psi_0\rangle = -t|\Psi_0\rangle$$

Two-electron tight-binding (Hückel) dimer

$$\hat{h} = -t \sum_{\sigma=\uparrow,\downarrow} \left(\hat{c}^\dagger_{1\sigma}\hat{c}_{2\sigma} + \hat{c}^\dagger_{2\sigma}\hat{c}_{1\sigma} \right) = \hat{h}^\dagger$$



$$|\Psi_0\rangle = \frac{1}{2}\left(\hat{c}^\dagger_{1\uparrow} + \hat{c}^\dagger_{2\uparrow}\right)\left(\hat{c}^\dagger_{1\downarrow} + \hat{c}^\dagger_{2\downarrow}\right)|\text{vac}\rangle \quad \rightarrow \quad \hat{h}|\Psi_0\rangle = -2t|\Psi_0\rangle$$

Order parameter

used to describe (unconventional) superconductivity in a material

$$d_{12} = \langle \hat{c}_{1\uparrow}\hat{c}_{2\downarrow} + \hat{c}_{2\uparrow}\hat{c}_{1\downarrow} \rangle$$

notation

$$\langle \hat{A} \rangle = \langle \Psi | \hat{A} | \Psi \rangle$$

Superconducting tight-binding (Hückel) dimer

$$\hat{h} = -t \sum_{\sigma=\uparrow,\downarrow} \left(\hat{c}^{\dagger}_{1\sigma}\hat{c}_{2\sigma} + \hat{c}^{\dagger}_{2\sigma}\hat{c}_{1\sigma} \right)$$

$$-D\left(\hat{c}_{1\uparrow}\hat{c}_{2\downarrow} + \hat{c}_{2\uparrow}\hat{c}_{1\downarrow} + \left(\hat{c}_{1\uparrow}\hat{c}_{2\downarrow} + \hat{c}_{2\uparrow}\hat{c}_{1\downarrow}\right)^{\dagger} \right) = \hat{h}^{\dagger}$$

Superconducting tight-binding (Hückel) dimer

$$\hat{h} = -t \sum_{\sigma=\uparrow,\downarrow} \left(\hat{c}^\dagger_{1\sigma}\hat{c}_{2\sigma} + \hat{c}^\dagger_{2\sigma}\hat{c}_{1\sigma} \right)$$

$$-D\left(\hat{c}_{1\uparrow}\hat{c}_{2\downarrow} + \hat{c}_{2\uparrow}\hat{c}_{1\downarrow} + \left(\hat{c}_{1\uparrow}\hat{c}_{2\downarrow} + \hat{c}_{2\uparrow}\hat{c}_{1\downarrow}\right)^\dagger\right) = \hat{h}^\dagger$$

$$|\Psi_0\rangle = \frac{1}{2\sqrt{t^2+D^2}} \Big[t\left(\hat{c}^\dagger_{1\uparrow}\hat{c}^\dagger_{1\downarrow} + \hat{c}^\dagger_{2\uparrow}\hat{c}^\dagger_{2\downarrow}\right)|\text{vac}\rangle + \sqrt{t^2+D^2}\left(\hat{c}^\dagger_{1\uparrow}\hat{c}^\dagger_{2\downarrow} + \hat{c}^\dagger_{2\uparrow}\hat{c}^\dagger_{1\downarrow}\right)|\text{vac}\rangle$$

$$-D\left(|\text{vac}\rangle - \hat{c}^\dagger_{1\uparrow}\hat{c}^\dagger_{1\downarrow}\hat{c}^\dagger_{2\uparrow}\hat{c}^\dagger_{2\downarrow}|\text{vac}\rangle\right) \Big]$$

$$\hat{h}\,|\Psi_0\rangle = -2\sqrt{t^2+D^2}\,|\Psi_0\rangle$$

Superconducting tight-binding (Hückel) dimer

$$\hat{h} = -t \sum_{\sigma=\uparrow,\downarrow} \left(\hat{c}^{\dagger}_{1\sigma}\hat{c}_{2\sigma} + \hat{c}^{\dagger}_{2\sigma}\hat{c}_{1\sigma} \right)$$

$$-D\left(\hat{c}_{1\uparrow}\hat{c}_{2\downarrow} + \hat{c}_{2\uparrow}\hat{c}_{1\downarrow} + \left(\hat{c}_{1\uparrow}\hat{c}_{2\downarrow} + \hat{c}_{2\uparrow}\hat{c}_{1\downarrow}\right)^{\dagger}\right) = \hat{h}^{\dagger}$$

$$|\Psi_0\rangle = \frac{1}{2\sqrt{t^2+D^2}}\Big[t\left(\hat{c}^{\dagger}_{1\uparrow}\hat{c}^{\dagger}_{1\downarrow} + \hat{c}^{\dagger}_{2\uparrow}\hat{c}^{\dagger}_{2\downarrow}\right)|\mathrm{vac}\rangle + \sqrt{t^2+D^2}\left(\hat{c}^{\dagger}_{1\uparrow}\hat{c}^{\dagger}_{2\downarrow} + \hat{c}^{\dagger}_{2\uparrow}\hat{c}^{\dagger}_{1\downarrow}\right)|\mathrm{vac}\rangle$$

$$-D\left(|\mathrm{vac}\rangle - \hat{c}^{\dagger}_{1\uparrow}\hat{c}^{\dagger}_{1\downarrow}\hat{c}^{\dagger}_{2\uparrow}\hat{c}^{\dagger}_{2\downarrow}|\mathrm{vac}\rangle\right)\Big]$$

$$\hat{h}|\Psi_0\rangle = -2\sqrt{t^2+D^2}\,|\Psi_0\rangle$$

Quasiparticle energy

Bogoliubov transformation

$$\hat{h} = -t \sum_{\sigma=\uparrow,\downarrow} \left(\hat{c}^\dagger_{1\sigma} \hat{c}_{2\sigma} + \hat{c}^\dagger_{2\sigma} \hat{c}_{1\sigma} \right)$$

$$-D \left(\hat{c}_{1\uparrow} \hat{c}_{2\downarrow} + \hat{c}_{2\uparrow} \hat{c}_{1\downarrow} + \left(\hat{c}_{1\uparrow} \hat{c}_{2\downarrow} + \hat{c}_{2\uparrow} \hat{c}_{1\downarrow} \right)^\dagger \right) = \hat{h}^\dagger$$

$$= \begin{bmatrix} \hat{c}^\dagger_{1\uparrow} & \hat{c}^\dagger_{2\uparrow} & \hat{c}_{1\downarrow} & \hat{c}_{2\downarrow} \end{bmatrix} \begin{bmatrix} 0 & -t & 0 & +D \\ -t & 0 & +D & 0 \\ 0 & +D & 0 & +t \\ +D & 0 & +t & 0 \end{bmatrix} \begin{bmatrix} \hat{c}_{1\uparrow} \\ \hat{c}_{2\uparrow} \\ \hat{c}^\dagger_{1\downarrow} \\ \hat{c}^\dagger_{2\downarrow} \end{bmatrix}$$

Bogoliubov transformation

$$\hat{h} = \begin{bmatrix} \hat{c}^\dagger_{1\uparrow} & \hat{c}^\dagger_{2\uparrow} & \hat{c}_{1\downarrow} & \hat{c}_{2\downarrow} \end{bmatrix} \begin{bmatrix} 0 & -t & 0 & +D \\ -t & 0 & +D & 0 \\ 0 & +D & 0 & +t \\ +D & 0 & +t & 0 \end{bmatrix} \begin{bmatrix} \hat{c}_{1\uparrow} \\ \hat{c}_{2\uparrow} \\ \hat{c}^\dagger_{1\downarrow} \\ \hat{c}^\dagger_{2\downarrow} \end{bmatrix}$$

$$\| $$

$$\underline{\underline{\mathcal{U}}} \begin{bmatrix} -\sqrt{t^2+D^2} & 0 & 0 & 0 \\ 0 & +\sqrt{t^2+D^2} & 0 & 0 \\ 0 & 0 & +\sqrt{t^2+D^2} & 0 \\ 0 & 0 & 0 & -\sqrt{t^2+D^2} \end{bmatrix} \underline{\underline{\mathcal{U}}}^\dagger$$

$$\underline{\underline{\mathcal{U}}} = \frac{1}{\sqrt{2\left(t^2+D^2\right)}} \begin{bmatrix} +t & +D & -t & +D \\ +\sqrt{t^2+D^2} & 0 & +\sqrt{t^2+D^2} & 0 \\ -D & +t & +D & +t \\ 0 & +\sqrt{t^2+D^2} & 0 & -\sqrt{t^2+D^2} \end{bmatrix}$$

Bogoliubov transformation

$$\hat{h} = \begin{bmatrix} \hat{d}^\dagger_{1\gamma} & \hat{d}^\dagger_{2\delta} & \hat{d}^\dagger_{2\gamma} & \hat{d}^\dagger_{1\delta} \end{bmatrix} \begin{bmatrix} -\sqrt{t^2+D^2} & 0 & 0 & 0 \\ 0 & +\sqrt{t^2+D^2} & 0 & 0 \\ 0 & 0 & +\sqrt{t^2+D^2} & 0 \\ 0 & 0 & 0 & -\sqrt{t^2+D^2} \end{bmatrix} \begin{bmatrix} \hat{d}_{1\gamma} \\ \hat{d}_{2\delta} \\ \hat{d}_{2\gamma} \\ \hat{d}_{1\delta} \end{bmatrix}$$

$$\begin{bmatrix} \hat{c}^\dagger_{1\uparrow} & \hat{c}^\dagger_{2\uparrow} & \hat{c}_{1\downarrow} & \hat{c}_{2\downarrow} \end{bmatrix} \underline{\underline{\mathscr{U}}}$$

$$\frac{1}{\sqrt{2\left(t^2+D^2\right)}} \begin{bmatrix} \frac{+t}{+\sqrt{t^2+D^2}} & \frac{+D}{0} & \frac{-t}{+\sqrt{t^2+D^2}} & \frac{+D}{0} \\ \frac{-D}{0} & \frac{+t}{+\sqrt{t^2+D^2}} & \frac{+D}{0} & \frac{+t}{-\sqrt{t^2+D^2}} \end{bmatrix}$$

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